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APPLICATION OF A NEURAL NETWORKS TO THE SLIDING OBSERVER OF THE MOBILE WHEELED-ROBOT VELOCITY *

Summary. *In the following research the analysis of the estimation of escape velocities of the mobile 2-wheeled robot is based on a measurement of angles of wheels' rotation. For the estimation of escape velocities of the wheels and the frame, the slide observer was introduced. The observer was equipped with a recurrent neural net, which exemplify a new depiction of the solution to the problem of estimating a state of mobile wheeled-robots. In the solution proposed, weights of the nets are estimated on-line, without an initial preparation. The accurate knowledge about non-linear character of an object or linear character of the system is not required due to unknown parameters. Theoretic deliberations, presented in the following research, were exemplified by simulate tests made in Matlab™.*

1. *Introduction*

Angles of wheels' self-rotation and angles of steered wheels turn in mobile wheeled-robots can be measured with relative precision. However, measurement of the escape velocities is usually influenced by measurement static, which leads to lower precision of keep up steering of the systems. In order to decrease the measurement interference, process of measured signal filtration is used, as a typical solution. This process usually causes phase displacement a change of the measured signal amplitude, which lowers precision of steering robots' motions in consequence. The following research suggests the slide observer, in which recurrent neutron net is introduced, as an alternative solution to the problem of estimating escape velocities of a mobile 2-wheeled robot. The fusion of the mobile wheeled-robot state observer is a complex problem for these objects are not linear, nonholonomic but multidimensional systems. Problems of estimation of non-linear systems were analysed in the researches [6,7,9,12]. Possibilities of wide application of neural nets, result from their characteristics and features, make them particularly interesting to be used for identifying and control non-linear systems [5,8,10,11]. Attempts to use neural nets in adaptive state-observers are also mad [6,12]. In the following research, to solve the problem of estimation of the angular velocities of the mobile 2-wheeled robot, the adaptation observer equipped with the recurrent neural net was used, which exemplify a new depiction of the solution to the problem of estimating a state of mobile wheeled-robots. The results obtained after introduced observer simulation follows that inaccuracy in estimation of mobile 2-wheeled-robot state is little and estimated weights of the neural net are limited. This research continues prior researches of the authors,

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concerning robust and adaptive observers of mobile wheeled-robots [1,2,4]. The remainder of the paper is organised as follows. Dynamic equations of the mobile 2-wheeled-robot motion are included in Chapter 2. Chapter 3 displays the process of state observer fusion, based on the Lapunov's theory. Chapter 4 includes results of observer tests, obtained after numerical simulation. Chapter 5 resumes the results of the research.

2. Dynamic equations of the mobile 2-wheeled-robot motion

We are going to consider the problem of mobile 2-wheeled-robot angular velocity-observer, diagram of which is shown in Fig. 1

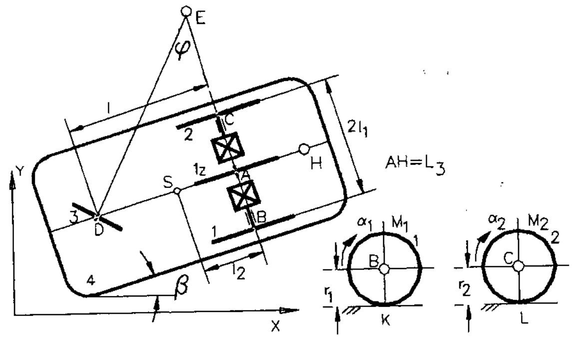

Fig. 1. Diagram of the mobile robot

and the dynamic equations of robot motions are given in limited configuration space [3,14]

$$M(a)\ddot{q}_2 + C(a,\dot{q}_2)\dot{q}_2 + F(a_F) = D(a)M_n \tag{1}$$

where matrixes and vectors have the following from:

$$q_2 = [\beta, \alpha]^T, \qquad M(a) = \begin{bmatrix} a_1 & 0 \\ 0 & a_2 \end{bmatrix}, \qquad C(a,\dot{q}_2) = \begin{bmatrix} 0 & -a_3\dot{\beta} \\ a_3\dot{\beta} & 0 \end{bmatrix},$$

$$F(a_F) = [a_4 - a_5, a_5 + a_4]^T, \qquad D(a) = \begin{bmatrix} a_6 & -a_6 \\ 1 & 1 \end{bmatrix}, \qquad M_n = [M_1, M_2]^T.$$

Established variables equal: q_2 is a vector of generalised displaces of-angle of the frame turn β - angle of the self-turn of the substitute propel wheel α , $M(a)$ is a matrix of inertia, $C(\dot{q}_2, a)\dot{q}_2$ is a vector of centrifugal and Coriolis's forces, $F(a_F)$ is a vector of motion resistance, $D(a)$ is a matrix of amplifications, M_n is a vector of the moments propelling driving wheels, a is a vector of the mobile robot parameters, which results from the system geometry, weights distribution and motions resistance and is defined by the following equation: $a_1 = 2m_1 l_1^2 + m_4 l_2^2 + I_s + 2I_{x1} + 2I_{z1}h_1$, $a_2 = 2I_{z1} + (2m_1 + m_4)r^2$, $a_3 = m_4 l_2 r$, $a_4 = N_1 f_1, a_5 = N_2 f_2$, $a_6 = h_1$, where $m_1 = m_2$, m_4 - substitute mass of wheels 1 and

2 and the frame, $I_{x1} = I_{x2}$ -substitute mass-inertia moments of the wheels 1 and 2 relative to the axes x_1 and x_2 which are connected with the wheels, $I_{z1} = I_{z2}$ - substitute mass- inertia moments of the adequate wheels relative to the axes of self-turn of the wheels. It was assumed that the axes of reference system connected with part "i" are the main central axes of inertia, however N_1, N_2 are pressure forces of the wheels 1 and 2, f_1, f_2 are turn friction coefficients of adequate wheels. l, l_1, l_2, $h_1 = r_1^{-1} l_1$ are adequate distances which result from the geometry of the system, $r_1 = r_2 = r$ are radiuses of adequate wheels.

Dynamic equations of motion (1) can be written in state space. Establishing $x_1 = q_2 \in R^n, x_2 = \dot{q}_2 \in R^{n_1}$, n = n₁ = 2 we receive

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= B[f(x) + G(x)u] \\ y &= x_1 \end{aligned} \tag{2}$$

where the matrixes and the vectors result from the equation (1).

3. Observer design

The problem of observing the state of the mobile robot is going to be solved with use of the sliding observer, the basics of which were introduced in the research [12]. The mobile robot's state observer specification is going to be assumed in the form of the following.

$$\begin{aligned} \dot{\hat{x}}_1 &= \hat{x}_2 - \Gamma_1 \tilde{x}_1 - K_1 \, sgn \, \tilde{x}_1 \\ \dot{\hat{x}}_2 &= -\Gamma_2 \tilde{x}_1 - K_2 \, sgn \, \tilde{x}_1 + B\left[\hat{f}(\hat{x}) + \hat{G}(\hat{x})u\right] \end{aligned} \tag{3}$$

where $\hat{x}_i$ is the estimate of state x_i, $\Gamma_1, \Gamma_2, K_1, K_2$ are positively specified matrixes. Characterising the signification of state estimation inaccuracy as:
$\tilde{x}_1 = \hat{x}_1 - x_1$, $\tilde{x}_2 = \hat{x}_2 - x_2$, and subtracting the system equation (2) from the observer equation (3) we get specification of the system, included in errors space.

$$\begin{aligned} \dot{\tilde{x}}_1 &= \tilde{x}_2 - \Gamma_1 \tilde{x}_1 - K_1 \, sgn \, \tilde{x}_1 \\ \dot{\tilde{x}}_2 &= -\Gamma_2 \tilde{x}_1 - K_2 \, sgn \, \tilde{x}_1 + B\left[\tilde{f}(x,\hat{x}) + \tilde{G}(x,\hat{x})u\right] \end{aligned} \tag{4}$$

where
$$\tilde{f}(x,\hat{x}) = \hat{f}(\hat{x}) - f(x) \quad \tilde{G}(x,\hat{x}) = \hat{G}(\hat{x}) - G(x), \tag{5}$$

$$\begin{aligned} K_1 \, sgn(\tilde{x}_1) &= [k_{11} \, sgn(\tilde{x}_{11}), k_{12} \, sgn(\tilde{x}_{12})]^T, \; k_{11}, k_{12} > 0 \\ K_2 \, sgn(\tilde{x}_1) &= [k_{21} \, sgn(\tilde{x}_{11}), k_{22} \, sgn(\tilde{x}_{12})]^T, \; k_{21}, k_{22} > 0 \end{aligned} \tag{6}$$

If the trajectories of the system of equations (4) meet initial requirements,

$$\begin{aligned} &\tilde{x}_{2i} - \gamma_{1i} \tilde{x}_{1i} - k_{1i} < 0 \quad dla \; \tilde{x}_{1i} > 0 \\ &\tilde{x}_{2i} - \gamma_{1i} \tilde{x}_{1i} + k_{1i} > 0 \quad dla \; \tilde{x}_{1i} < 0 \quad i = 1,2 \end{aligned} \tag{7}$$

the are convergent on the surface $\tilde{x}_1 = 0$. The motion of the system on the sliding surface is specified by the equation (6) resulted from the equations (4) and (5)

$$\dot{\tilde{x}}_2 = -K\tilde{x}_2 + B\left[\tilde{f}(x,\hat{x}) + \tilde{G}(x,\hat{x})u\right] \tag{8}$$

Further analysis assumes that

$$K_2 K_1^{-1} = K = kI \tag{9}$$

According to approximate characteristic of the neural nets [10,13] and assuming constants weights of the first layer of the net, given function in configuration (2) can be written in the form of

$$f(x) = W_f^T S_f(x) + \varepsilon_f(x), \qquad G(x) = \left[W_{G1}^T, W_{G2}^T\right] S_G(x) + \varepsilon_G(x) \tag{10}$$

where W_f is the matrix of dimensioned weights of the net (m_f x n_1), m_f is the number of the basis functions in the net, $S_f(x)$ is a m_f dimensioned vector of the basis functions, W_{G1}, W_{G2} are matrixes of the dimensions equal to m_g x n_1 , $S_G(x)$ is a m_g dimensioned vector of the basis functions, $\varepsilon_f(x), \varepsilon_G(x)$ are inaccuracies of approximation of the function (10) that meets configuration $\|\varepsilon_f(x)\| \le \varepsilon_f, \|\varepsilon_G(x)\| \le \varepsilon_G$. Assuming estimates of the function (10) in the form of

$$\hat{f}(\hat{x}) = \hat{W}_f^T S_f(\hat{x}), \qquad \hat{G}(\hat{x}) = \left[\hat{W}_{G1}^T, \hat{W}_{G2}^T\right] S_G(\hat{x}) \tag{11}$$

errors in approximation of the function (5) can be written as

$$\tilde{f}(x,\hat{x}) = \tilde{W}_f^T S_f(\hat{x}) + c_f(t) - \varepsilon_f(x), \qquad \tilde{G}(x,\hat{x}) = \left[\tilde{W}_{G1}^T, \tilde{W}_{G2}^T\right] S_G(\hat{x}) + c_G(t) - \varepsilon_G(x) \tag{12}$$

were $\tilde{W}_f, \left[\tilde{W}_{G1}, \tilde{W}_{G2}\right] = \tilde{W}_G$ are errors of weights estimation, $c_f(t) = W_f^T \tilde{S}_f(x,\hat{x})$, $c_G(t) = W_G^T \tilde{S}_G(x,\hat{x})$ are additional signals of inaccuracies in approximating, due to errors of net, output signals. According to the foregoing analysis, equation (8) can be written in the form of

$$\dot{\tilde{x}}_2 = -K\tilde{x}_2 + B\left[\tilde{W}_f^T S_f(\hat{x}) + \tilde{W}_G S_\Gamma(\hat{x})\right] + BR \tag{13}$$

where $S_\Gamma(\hat{x}) = u \otimes S_G(\hat{x})$, $R = c_f(t) - \varepsilon_f(x) + \left[c_G(t) - \varepsilon_G(x)\right]u$, $\otimes$ is the Kronecker's product. In order to calculate the law of learning weights of the neural net, which is essential for the net's stability, and estimate the convergence of the output net's signals on the required quantities, we introduce the Lapunov's function in the form of:

$$V = 0.5\tilde{x}_2^T P\tilde{x}_2 + 0.5tr\tilde{W}_f^T F_f^{-1}\tilde{W}_f^T + 0.5tr\tilde{W}_G^T F_G^{-1}\tilde{W}_G^T \tag{14}$$

$F_f = F_f^T > 0$, $F_G = F_G^T > 0$ are the characteristics of the matrixes enclosed in the configuration (14), and for the matrix, K that results from the equation (13), is exponentially stable, there are the matrixes $P = P^T > 0$, $Q = Q^T > 0$, existing, due to which the Lapunov's equation $K^T P + PK + Q = 0$ is completed, making a differential calculus along the result of the system (8), we get:

$$\dot{V} = -\tilde{x}_2^T Q\tilde{x}_2 + x_2^T PB\left[\tilde{W}_f^T S_f(\hat{x}) + \tilde{W}_G^T S_\Gamma(\hat{x}) + R\right] + tr\tilde{W}_f^T F_f^{-1}\dot{\tilde{W}}_f^T + tr\tilde{W}_G^T F_G^{-1}\dot{\tilde{W}}_G^T \tag{15}$$

Considering the ideal situation, where R=0, and selecting the principle of learning weights of the net

$$\dot{\hat{W}}_f = -F_f S_f(\hat{x})\tilde{x}_2^T PB \, , \, \dot{\hat{W}}_G = -F_G S_\Gamma(\hat{x})\tilde{x}_2^T PB \tag{16}$$

we finally get

$$\dot{V} = -\tilde{x}_2^T Q \tilde{x}_2 \tag{17}$$

In the equation (16) there is a signal of not-measured quantities $\tilde{x}_2$. This signal can be calculated indirectly in the equation (4) where $\tilde{x}_1 = 0, \tilde{x}_2 = K_1 \, sgn(\tilde{x}_1)$. The adaptation law (16) guarantees the following characteristics: $lim_{t\to\infty} \tilde{x}_2 = 0$, $lim_{t\to\infty} \dot{\hat{W}}_f = 0$, $lim_{t\to\infty} \dot{\hat{W}}_G = 0$.

4. Simulation results

In order to verify the observer suggested, the numeric experiment was carried out on the assumption that the mobile robot shown in the Fig.1 carries an unknown mass Δm_4. Assuming the quantities of moments driving the wheels, simple dynamic problem was calculated, and gave the angle velocities of the frame $\dot{\beta}$ and driving wheel $\dot{\alpha}$. In the simulation it was assumed that the system, were $t \geq 2[s]$, was disrupted by some parameters resulted from mass increase of the frame m_4 equals $\Delta m_4 = 250$ [kg]. Çalculations were carried out for the following number of the basis functions: $m_f = 6$, $m_g = 3$. The rest of the data, and the average quantities of the parameters, which were assumed for the simulation, are given below.

$$\begin{array}{llllll}
a_1 = 620.13 & a_2 = 22.47 & a_3 = 87.6 & a_4 = 75.0 & a_5 = 75.0 & a_6 = 2.33 \\
k_{11} = 0.1 & k_{12} = 0.1 & k_{21} = 0.2 & k_{22} = 0.2 & \beta(0) = 0.0 & \dot{\beta}(0) = 0.0 \\
\hat{\beta}(0) = 0.0 & \dot{\hat{\beta}}(0) = 0.0 & \alpha(0) = 0.0 & \dot{\alpha}(0) = 0.0 & \hat{\alpha}(0) = 0.0 & \dot{\hat{\alpha}}(0) = 0.0
\end{array}$$

Every quantity given is characterised by SI system units. Fig.2 shows the structure of the observer that was assumed.

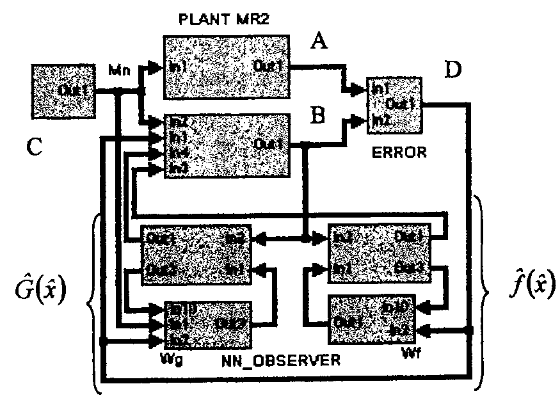

Fig.2. Calculate structure of the observer.

Suggested structure of the observer is made up of six basic units. The aim of unit "C" is to generate constant driving moments of the wheels 1 and 2 of the mobile robot. Unit "A" generates kinetic parameters of the mobile robot, based on the equation (2). Dynamic structure of the observer (3) is realised in unit "B". In unit "D", the errors of the observer are calculated. On the basis of the errors, using the neural net, the signals that compensate dynamic non-linearity of the mobile-robot are calculated. The law of nets' weights adaptation (16) is realised in units W_f and W_g. The first experimental was carried on the assumption that the mobile-robot moved along a straight trajectory and an appearance of the parametric uncertainties. In the simulation the following quantities of the driving moments were assumed: $M_1 = 100[Nm]$ and $M_2 = 100[Nm]$. Fig. 3 shows time course of the angle velocity of wheel 1z $\dot{\alpha}$ generated in unit "PLANT MR2". In this case angle velocity of the frame $\dot{\beta}$ equals zero. Error in estimation of velocity $e_2 = \dot{\hat{\alpha}} - \dot{\alpha}$ is shown in Fig. 3b.

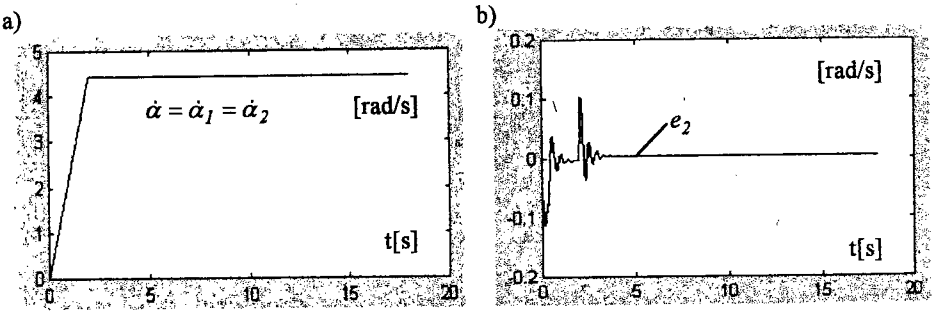

Fig. 3b. Assigned velocity $\dot{\alpha}$ and its estimation error.

Angular velocities of wheels 1 and 2 were calculated in by equation $\dot{\alpha}_1 = \dot{\alpha} + \dot{\beta}a_6$, $\dot{\alpha}_2 = \dot{\alpha} - \dot{\beta}a_6$. The second experiment was carried on the assumption that the mobile-robot moved along a curve trajectory and an appearance of the parametric uncertainties. In the simulation the following quantities of the propelling moments were assumed: $M_1 = 100[Nm]$ i $M_2 = 99[Nm]$. Picture 4a and 4b shows time course of the angle velocities of wheel 1z $\dot{\alpha}$, wheel 1 $\dot{\alpha}_1$, wheel 2 $\dot{\alpha}_2$, and frame $\dot{\beta}$, generated in unit "PLANT MR2".

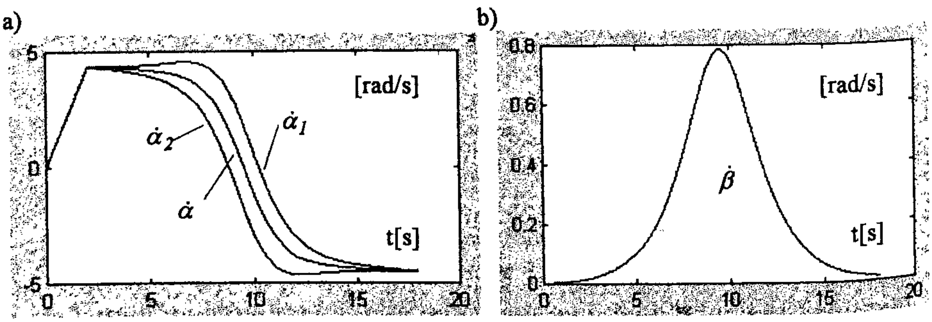

Fig.4. Generated angular velocities.

Calculations of the inaccuracies of the observer are shown in Fig. 5. Fig. 6a exemplifies courses of selected estimates of the nets' weights, and Fig. 6b exemplifies inaccuracies of estimating angle velocity of substitute wheel excepting the dynamic characteristic of the mobile-robot in the observer structure.

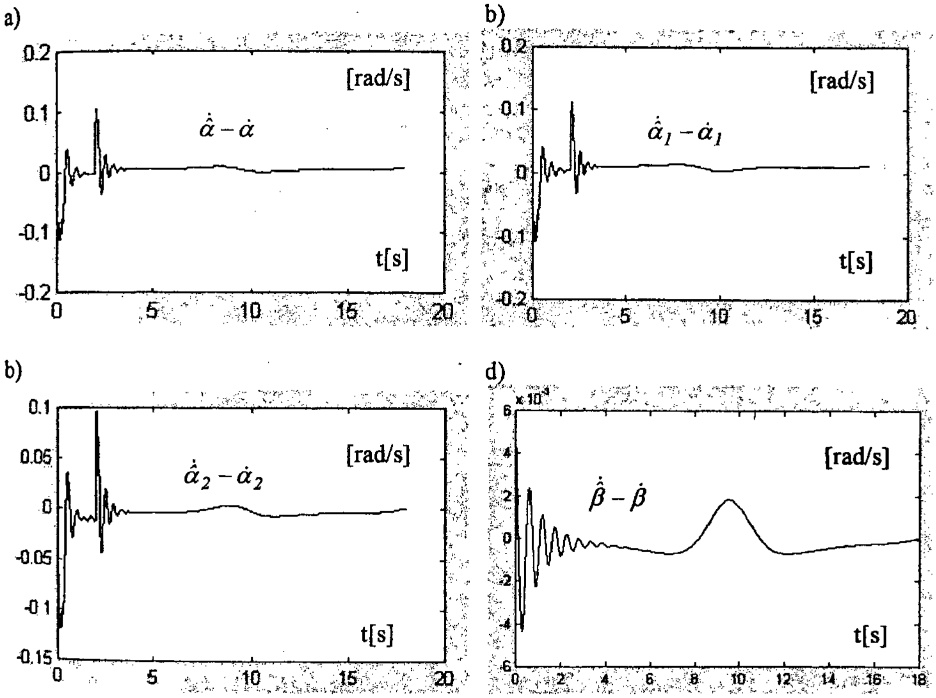

Fig. 5. Errors of the observer.

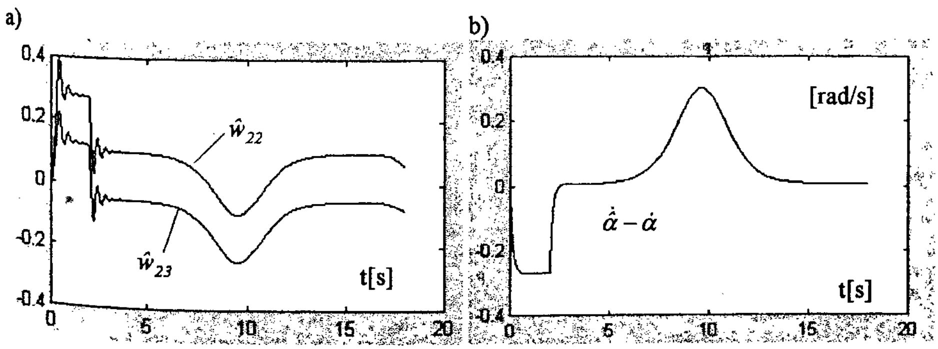

Fig.6a. Estimates of selected nets' weights. (b). Error of the observer in the appearance of non-parametric disturbances.

On the basis of the results, it can be claimed that the errors of the estimated angle velocities are small. Furthermore, basic features of the solution, resulted from configurations (16) and (17) are met.

5. Conclusions

In the foregoing research the problem of the synthesis of adaptive angle-velocities observer of the mobile-2 wheeled-robot was analysed. It was assumed that model's uncertainties that occurred, resulted from parametric interference. The structure of suggested observer, uses characteristics of sliding motion of systems with variable structure and approximate characteristics of neural nets. In the solution suggested, for approximation of the vector and signal matrix, two neural nets were introduced. The weights of the nets are estimated on line, without initial process of learning. Accurate knowledge about non-linearity of the system is not required for the parameters are unknown. Final solution bases on the assumption, that real quantities of weights, thanks to which we can present perfectly non-linearity of the object, are known. Taking this additional interference into consideration demands modification of established structure of the observer, by introducing an additional signal that compensates inaccuracies of the established structure. As a result of this modification we can get a robust solution that leads to increasing precision in observing signals.

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