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REFERENCE MODEL OF CONSTRAINT SATISFACTION PROBLEM DECOMPOSITION

An analysis of relations between the decision problem and the solution method facilitates adopting a model connecting the problem specification including the implementation restrictions of the available methods. The proposal of such a platform is presented in the reference model of the constraint satisfaction problem (CSP) decomposition. The reference model allows to analyse possible constraint satisfaction problem decompositions and many searching strategies within the decompositions.

MODEL REFERENCYJNY DEKOMPOZYCJI PROBLEMU SPEŁNIENIA OGRANICZEŃ

Analiza relacji pomiędzy problemem decyzyjnym a przyjętą metodą rozwiązania umożliwia przyjęcie modelu łączącego specyfikację (potrzeby) problemu z implementacyjnymi ograniczeniami dostępnych metod. Propozycję takiego rozwiązania przedstawia model referencyjny dekompozycji problemu spełniania ograniczeń (PSO). Przedstawiony model referencyjny pozwala analizować dopuszczalne dekompozycje PSO oraz strategie poszukiwania rozwiązania w ramach tych dekompozycji.

1. INTRODUCTION

Due to the prevailing unique character of work orders in SME (small and medium enterprises) it is necessary to be able to evaluate quickly and precisely the possibility of balancing production capacity of a company with the requirements set in an agreement with the employer.

Searching for feasible solutions, regarding for example resources allocation, time lags, makespan, costs, etc, has to be preceded by formulation of a feasibility problem or equivalently by a constraint satisfaction problem (CSP). Moreover, solution to a makespan-feasible problem permits a user to investigate the effect of a new work order impact on the performance of a manufacturing system. In other words, it enables finding an answer to the most important question whether a given work order can be accepted to be processed in the manufacturing system, i.e., whether its completion time, batch size, and its delivery period satisfy the customer requirements while satisfying constraints imposed by the enterprise configuration and the process of manufacturing of other products [1].

Usually the first solution meeting a set of constraints which link decision variables specifying the manufacturer's possibilities is sought. The variables characterize conditions of the execution of a work order and decision variables in the relation manufacturer-consumer. Decisions are usually formed as a *Constraint Satisfaction Problem.* It specifies a set of decision variables with values which are included in finite discreet domains; variables of different nature and character, from periods of resources availability on, through the volumes of production and transportation batches, to the deadlines and taking-over prices of the specific batches of the order.

The considered problem specification determines the choice of the appropriate tool (method and its implementation) used for solving the problem. It is necessary to use the integrated platform (model) which enables to compare the specification of the solving model (its specification, decomposition) with the capabilities of every method and/or their computer implementation. Such model should allow choosing the appropriate method without high cost and risk of success realization.

2. CONSTRAINT SATISFACTION PROBLEM

From the available evaluations [2,3] it results that over 95% of all manufacturing and services decision problems are included in the Constraint Satisfaction Problems, for which many Constraint Programming (CP) languages were worked out (especially Constraint Logic Programming). The declarative character of CP languages (and their implementation) and a high efficiency in solving combinatorial problems creates an attractive alternative for the currently available (based on operation research techniques) systems of computer integrated management.

Consider the CSP that consists of a set of variables $X = \{x_1, x_2, ... ,x_n\}$, their domains $D = \{D_i \mid D_i = [d_{i1}, d_{i2}, ..., d_{ij}, ..., d_{im}], i = 1..n\}$, and a set of constraints of these variables $C = \{C_i \mid i = 1..L\}$ which reduces value of the decision variables. A solution is such a value assignment of the variables that all constraints are satisfied. Searching for the feasible solution (i.e. the variables value are due all of given constraints) or optimal solution (a set of solutions). This solution extremalizes the objective function specified on a subset of arbitrary chosen decision variables.

The following CSP notation is applied: CSP = ((X,D),C), where $c \in C$ is a constraint specified by a predicate $P[x_k,x_l,...,x_h]$ defined on a subset of the X set. The considered problem may be decomposed into subproblems.

For the illustration purposes lets us consider the following problem:

CSP = ((X,D),C) , where $X = \{x_1,x_2,...,x_{12}\}$, $D = \{D_1,D_2,...,D_{12}\}$, $C = \{c_1,c_2,...,c_8\}$, where: $c_1 = P_1[x_1,x_2,x_3]$, $c_2 = P2_1[x_2,x_4,x_5]$, $c_3 = P_3[x_4,x_6]$, $c_4 = P_4[x_7,x_8]$, $c_5 = P_5[x_4,x_7]$, $c_6 = P_6[x_9,x_{10}]$, $c_7 = P_7[x_8,x_9]$, $c_8 = P_8[x_{11},x_{12}]$. Two arbitrary chosen feasible CSP decompositions are presented in fig. 1. The subproblems that cannot be decomposed are side to be so called the elementary subproblems.

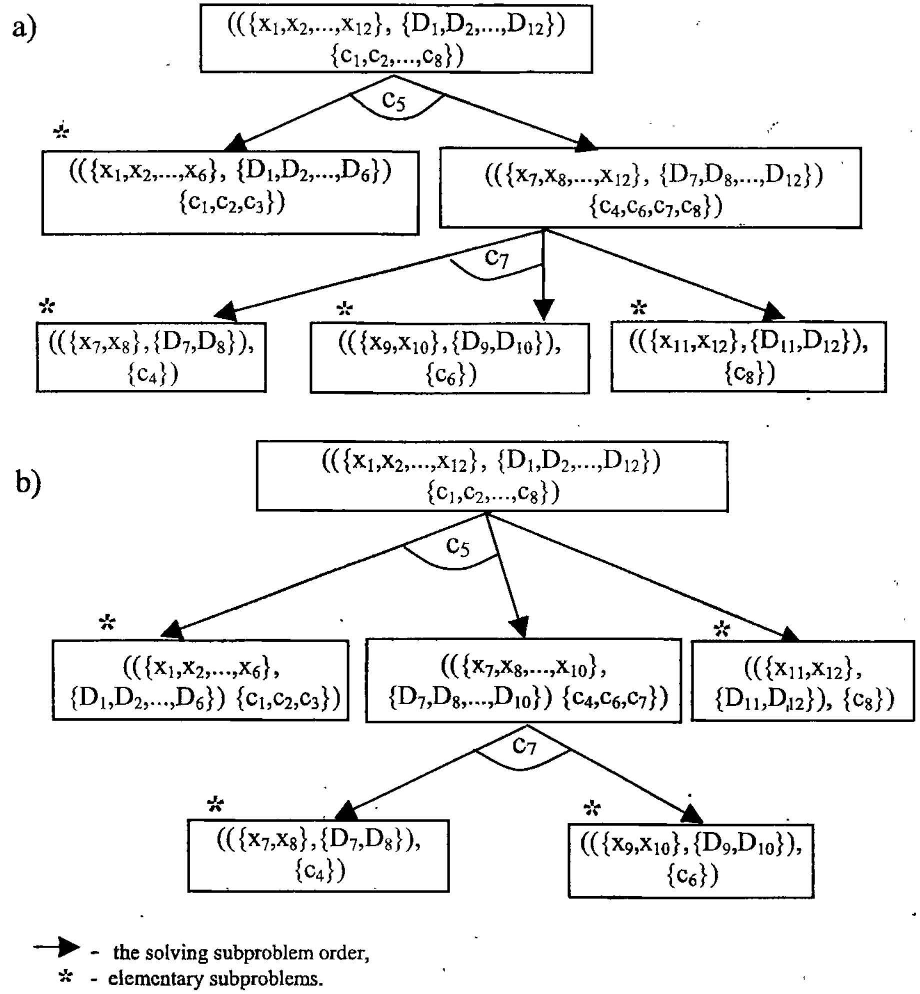


Fig. 1. The CSP feasible decompositions

The presented example illustrates the possibility of choosing the searching strategy that minimizes the number of potential backtrackings (related with the size of variables domains).

It is assumed that the searching strategies possible variants are subject to the principles of CSP decomposition (assumed in the previous step). They take into account programming system operators and the possibility of using the constraint propagation techniques.

For the given specification of the problem it is necessary to assort such a method which can solve this problems without introduced simplification. It means that this method has to be effective to solve decision problem which can be specified in agreement with this method. These observation implies the need to work out the reference model of constraint satisfaction problem decomposition. The model shall facilitate solving the following problems: given is constraint satisfaction problem; what implementation of the CP/CLP language facilitates its solution? (what searching strategy minimizes the number of potential backtrackings?).

3. REFERENCE MODEL

Given is CSP specification ((X,D),C) and a programming language CP/CLP i.e. with a standard functions library. The problem is based on the choosing the problem representation method and its programming systems (in CP languages) and a method of setting the searching strategies possible variants.

The problem representation and the potential (currently available) programming systems assume a possibility of CSP decomposing into subproblems. It facilitates describing the operators (functors) of programming language in categories of decision variables, their domains and constraints. The operators shall especially have the formula $F((X,D)) = F((x_a,D_a),(x_b,D_b),\ldots,(x_v,D_v))=\{(x_a,D_a),(x_b,D_b),\ldots,(x_v,D_v)\}$. For example the functor influencing an elementary problem shall result in a set of vectors with value of the problem decision variable domains.

The possible problem decompositions may be interpreted as appropriate searching strategies, determined by a specified number of subproblems and the sequence of solving them.

The following notation is applied:

$CSP = ((X,D),C)$ – the solving problem specification

$CSP^1_j=((X^1_j,D^1_j),C^1_j)$ – the j-th CSP subproblem specification (its decomposition)

$CSP^i_{j1,j2,\ldots ji}=((X^i_{j1,j2,\ldots ji},D^i_{j1,j2,\ldots ji}),C^i_{j1,j2,\ldots ji})$ – the ji-th subproblem specification, where $CSP^{i-1}_{j1,j2,\ldots ji-1}$ - its direct decomposition

$(\{CSP^{i+1}_{j1,j2,\ldots ji+1} | j_{i+1}\in \{1\ldots w\}\}, R^i_j\})$ – the graph's representation of the $CSP^i_{j1,j2,\ldots ji}$, problem decomposition, where $CSP^{i+1}_{j1,j2,\ldots ji+1}$- j_{i+1} – the CSP first direct decomposition $CSP^i_{j1,j2,\ldots ji}$, R^i_j – the set of relations join the direct decomposition $\{CSP^{i+1}_{j1,j2,\ldots ji+1}| j_{i+1}\in \{1\ldots w\}\}$, where $R^i_j=\{{}^{k,l}C^i_j \mid k,l\in\{1..w\}, k\neq l\}$ is a set of relations join the CSP^{i+1}_{jk} subproblem with CSP^{i+1}_{jl}.

Reference model of $CSP = ((X,D),C)$ decomposition refers to an object architecture decomposition $(\{CSP^{i+1}_{j1,j2,\ldots,ji+1} | j_{i+1}\in \{1\ldots w\}\}, R^i_j)\})$ meeting the following conditions:

For simplicity the notation of the first i-th indexes are omitted $_{j1,j2,\ldots,ji}$, so the notation CSP^{i+1}_{ji+1} stands for $CSP^{i+1}_{j1,j2,\ldots,ji+1}$.

i) $CSP^{i+1}_{ji}= ((X^{i+1}_{ji},D^{i+1}_{ji}),C^{i+1}_{ji})$ where:

$$X^{i+1}_{jr}= {}^rX^i_{ji}\ ;\ X^i_{ji} = {}^1X^i_{ji} \cup {}^rX^i_{ji} \cup\ldots\cup {}^wX^i_{j};$$

$$\forall r\in\{1,\ldots,w\} \mid {}^rX^i_{ji}\neq\varnothing$$

$$\forall r,u\in\{1,\ldots,w\} \mid r\neq u \Rightarrow {}^rX^i_{ji}\cap {}^uX^i_{ji} =\varnothing$$

$$D^{i+1}_{jr}= {}^rD^i_{ji}$$

$$C^{i+1}_{jr}= {}^rC^i_{ji};\ C^i_{ji}= {}^1C^i_{ji} \cup {}^rC^i_{ji} \cup\ldots\cup {}^wC^i_{ji}$$

$$\forall r\in\{1,\ldots,w\} \mid {}^rC^i_{ji}\neq\varnothing,$$

$$\sum_{r=1}^{w}\left|{}^{r}C^{i}_{ji}\right| = \left|C^{i}_{ji}\right|$$

$\forall r,u \in \{1,\dots,w\}\ \forall c \in {}^{r}C^{i}_{ji} \mid r \neq u \Rightarrow \phi(c) \cap {}^{u}X^{i}_{ji} = \varnothing$,

where $\phi(c) = \{x_a,x_b,x_v\}$ for $c = P[x_a,x_b,x_v]$

ii) $CSP^{i+1}_{ji} = ((X^{i+1}_{ji},D^{i+1}_{ji}),C^{i+1}_{ji})$ where:

$X^{i+1}_{jr} = {}^{r}X^{i}_{ji}$; $X^{i}_{ji} = {}^{1}X^{i}_{ji} \cup {}^{r}X^{i}_{ji} \cup \dots \cup {}^{w}X^{i}_{j}$;

$\forall r \in \{1,\dots,w\} \mid {}^{r}X^{i}_{ji} \neq \varnothing$

$\forall r,u \in \{1,\dots,w\} \mid r \neq u \Rightarrow {}^{r}X^{i}_{ji} \cap {}^{u}X^{i}_{ji} = \varnothing$

$D^{i+1}_{jr} = {}^{r}D^{i}_{ji}$

$C^{i+1}_{jr} = {}^{r}C^{i}_{ji}$; $C^{i}_{ji} = {}^{1}C^{i}_{ji} \cup {}^{r}C^{i}_{ji} \cup \dots \cup {}^{w}C^{i}_{ji} \cup {}^{k,l}C^{i}_{j}$;

$\forall r \in \{1,\dots,w\} \mid {}^{r}C^{i}_{ji} \neq \varnothing$, ${}^{k,l}C^{i}_{j} \neq \varnothing$

$$\sum_{r=1}^{w} |{}^{r}C^{i}_{ji}| + |{}^{k,l}C^{i}_{j}| = |C^{i}_{ji}|$$

$\forall r,u \in \{1,\dots,w\}\ \forall c \in {}^{r}C^{i}_{ji} \mid r \neq u \Rightarrow \phi(c) \cap {}^{u}X^{i}_{ji} = \varnothing$,

where $\phi(c) = \{x_a,x_b,x_v\}$ for $c = P[x_a,x_b,x_v]$

$\forall c \in R(C^{i}_{ji})\ \exists k,l \in \{1,\dots,w\} \mid \phi(c) \cap {}^{k}X^{i}_{ji} \neq \varnothing\ \&\ \phi(c) \cap {}^{l}X^{i}_{ji} \neq \varnothing$

For the illustration lets us consider the reference model graph's representation $(\{CSP^{i+1}_{j1,j2,\dots ji+1} \mid j_{i+1} \in \{1\dots w\}\}, R(C^{i}_{ji})\})$ of the given example (see 1.a). The following notation is applied:

$CSP=((\{x_1 \div x_{12}\},\{D_1 \div D_{12}\}),\{c_1 \div c_8\})$

$CSP \sim (\{CSP^1_1, CSP^1_2\}, R)$,

$CSP^1_1 = ((\{x_1,x_2,\dots,x_6\}, \{D_1, D_2,\dots, D_6\}), \{c_1,c_2,c_3\})$

$CSP^1_2 = ((\{x_7,x_8,\dots,x_{12}\}, \{D_7, D_8,\dots, D_{12}\}), \{c_4,c_6,c_7,c_8\})$

$R=\{{}^{1,2}C\}$; ${}^{1,2}C = \{c_5\}$

$CSP^1_2 \sim (\{CSP^2_{2,1}, CSP^2_{2,2}, CSP^2_{2,3}\}, R^1_2)$,

$CSP^2_{2,1} = (\{x_7,x_8\}, \{D_7, D_8\}),\{c_4\})$

$CSP^2_{2,2} = (\{x_9,x_{10}\},\{D_9, D_{10}\}),\{c_6\})$

$CSP^2_{2,2} = (\{x_{11},x_{12}\},\{D_{11}, D_{12}\}),\{c_8\})$

$R^1_2=\{{}^{1,2}C^1_2\}$; ${}^{1,2}C^1_2=\{c_7\}$

The considered reference model example is illustrated in fig. 2.

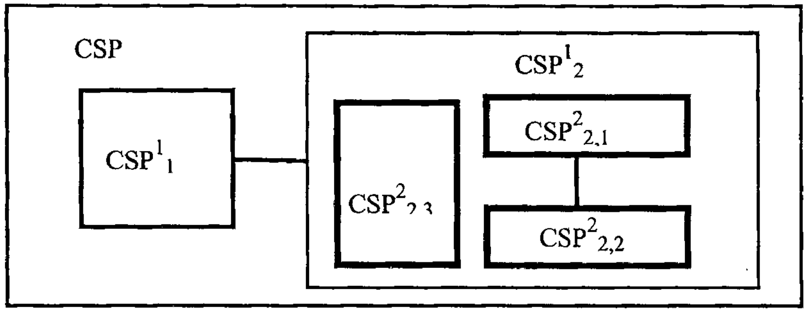


Fig. 2. The graphic illustration of the CSP objective instance decomposition

Links between objects mean that subproblems should be solved jointly. The presented instance of the CSP decomposition is one of the representation possibilities.

4. AND/ OR GRAPH CSP REPRESENTATION

CSP decompositions instances presented in fig. 1. and fig. 2. do not exhaust all potential decomposition possibilities.

Let us introduce decomposed subproblems notation: $CSP^{i}_{j',k,l}$ – represents l-th decomposition of the i-th problem (where $i=|\{j,k,l\}|$). This problem constitutes a k-th decomposition of an l-th problem which is a j-th decomposition of problem i-2-th which constitutes an i –2-th decomposition of the output CSP problem. According to his notation 'j'- represents a decompositions problem which are respectively mutually independent (i.e. appropriate subsets of variables are not linked by any constraints). In accordance with the presented notation CSP decomposition in fig. 1. a) includes subproblems marked in bold in fig. 3.

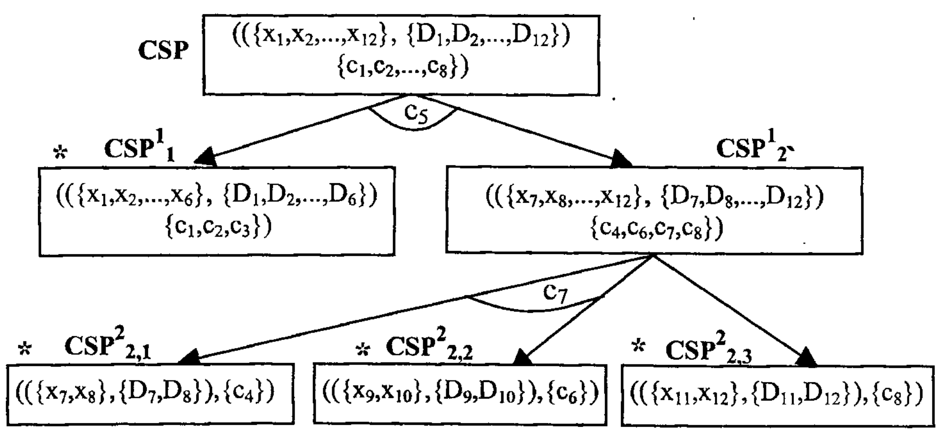


Fig. 3. CSP decomposition

The presented instance (fig. 3.) represents a decomposition presented by means of a CSP decomposition graph (fig. 4.). This graph facilitates an integrated, easy to be interpreted representation. Letters in notation of individual subproblems represent structures of the subproblems (decision variables, domains and constraints).

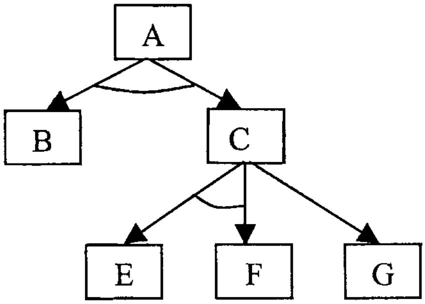
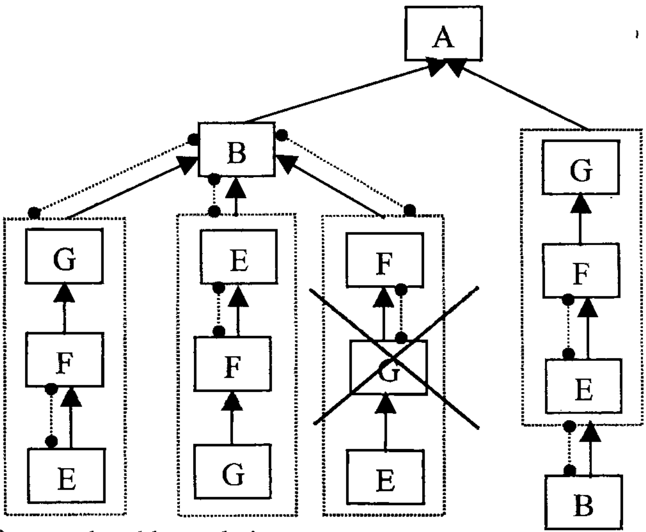


Fig. 4. Graph of a CSP decomposition

Fig. 5. illustrates an AND/OR graph of CSP decomposition. This graph presents alternative CSP solution searching strategies. It takes into account direct relations between subproblems.



- subproblem solutions sequence,
- relations between subproblem decompositions (AND notation),
- unfeasible searching strategy.

Fig. 5. AND/OR graph of CSP searching strategies

The AND/OR graph representation of acceptable CSP decompositions facilitates an analysis of all potential ways of solving a problem. It is easy to note that the AND/OR arches may be connected with the weighs which specify the number of necessary searches (domain items). This is the way of choosing a possible strategy variant e.g. with the smallest number of backtrackings. It means that the particular AND/OR strategies of graph representation may initially be varied according to the different criteria of searching efficiency. This in turn means a possibility of giving up time- and cost consuming simulation experiments.

5. CONCLUDING REMARKS

A CP modelling framework supporting decision making systems synthesis, which in turn are aimed at the design of a decision support system aimed at small and middle size enterprises is considered.

The reference model proposed allows to prototype different searching strategies of the production flow planning. Therefore, it allows one to estimate the searching strategy time just on the base of a given problem statement data structure, variable domains and a way of a problem decomposition.

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