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PRODUCTION FLOW PLANNING BASED ON A REFERENCE MODEL OF CONSTRAINT SATISFACTION PROBLEM DECOMPOSITION

Constraint programming (CP) is an emergent software technology for declarative description and effective solving of large combinatorial problems especially in areas of integrated production planning. In that context, the CP can be considered as a well-suited framework for development of decision making software supporting small and medium size enterprises in the course of Production Process Planning (PPP). The aim of the paper is to present the CP modeling framework as well as to illustrate its application to decision making in the case of a new production order evaluation. So, the contribution emphasizes benefits derived from CP-based DSS and focuses on constraint satisfaction driven decision-making rather than on an optimal solution searching

PLANOWANIE PRZEPŁYWU PRODUKCJI Z WYKORZYSTANIEM MODELU REFERENCYJNEGO DEKOMPOZYCJI PROBLEMU CSP

Programowanie z ograniczeniami (CP) jest nową techniką deklaratywnego opisu i efektywnego rozwiązywania dużych problemów kombinatorycznych szczególnie w obszarze zintegrowanego planowania produkcji. W tym kontekście, CP stanowi atrakcyjna platformę rozwoju oprogramowania wspomagającego planowanie procesów produkcyjnych w małych i średnich przedsiębiorstwach, w trybie na bieżąco.

1. INTRODUCTION

Constraint programming (CP) is an emergent software technology for declarative description and effective solving of large combinatorial problems especially in areas of integrated production planning. In that context, the CP can be considered as a well-suited framework for development of decision making software supporting small and medium size enterprises in the course of Production Process Planning (PPP). The aim of the paper is to present the CP modeling framework as well as to illustrate its application to decision making in the case of a new production order evaluation. So, the contribution emphasizes benefits derived from CP-based DSS and focuses on constraint satisfaction driven decision-making rather than on an optimal solution searching.

In that context, the CP can be considered as a well-suited framework for development of decision making software aimed at support of the small and medium size enterprises in the course of the Production Process Planning (PPP). Because of its declarative nature, for a use that is enough to state what has to be solved instead *how* to solve it [2].

The aim of the paper is to present the CP modelling framework as well as to illustrate its application to decision making in the case of a new production order evaluation (more precisely_production process planning). Finding an answer to the question whether a given work order can be accepted to be processed in the production system seems to be a fundamental from the customer-driven, and highly competitive market point of view. In that context decision making regards to the question whether enterprise's capability allows to fulfil constraints imposed by the production order requirements, i.e. whether its completion time, batch size, and its delivery period satisfy the customer requirements while satisfying constraints imposed by the enterprise configuration taking into account available resources, know how, experience, and so on. In the case of the response to this question being positive, i.e. there exist a way guaranteeing to complete a production order, the next question regards of finding of the most efficient one (e.g. as to be competitive on the market) [3].

The detailed illustrative example of the approach usage is provided as well as its implementation in Oz language.

2. REFERENCE MODEL OF CSP DECOMPOSITION

The element guaranteeing competitiveness of an enterprise on the market is its ability to make prompt and appropriate decisions relating to customer needs and production possibilities of the producer. Decision making problems occur, particularly often in small and medium-sized enterprises and are connected to acceptance of a new production order. Usually, the first solution, which satisfies the set of limits, is search. This set connects decision variables, which specify manufacturer abilities, variables that characterize order realization conditions and decision variable between consumer and manufacturer.

Decisions taken, are usually formulated in *Constraint Satisfaction Problem* (CSP) form, for which dedicated programming languages with constraints are elaborated (*Constraint Programming* CP), in particular *Constraint Logic Programming* CLP. Declarative character of CP languages and high efficiency of implemented decision aided packets creates an attractive alternative (enabling on-line work) to accessible computer integrated management systems [4, 5].

2.1 Constraint satisfaction problem

Let's consider the constraint satisfaction problem (CSP) formulated as follow: finite set of variables is given $X = \{x_1, x_2, \ldots, x_n\}$, family domains of variables $D = \{D_i \mid D_i = [d_{i1}, d_{i2}, \ldots, d_{ij}, \ldots, d_{im}], i = 1..n\}$ and finite set of constraints $C = \{C_i \mid i = 1..L\}$, which limits decision variables values.

Request is either admissible solution, that means solution in which values of all variables satisfy all constraints (one, soon obtained, either or all possible) or optimal solution (in general set of solutions) that extreme objective function definite on chosen decision variables subset.

Lets assume the following notation of constraints satisfaction problem:

$$CSP = ((X,D),C),$$

where: $c \in C$ is a certain predicative $P[x_k,x_l,\ldots,x_h]$ defined on a subset of set X.

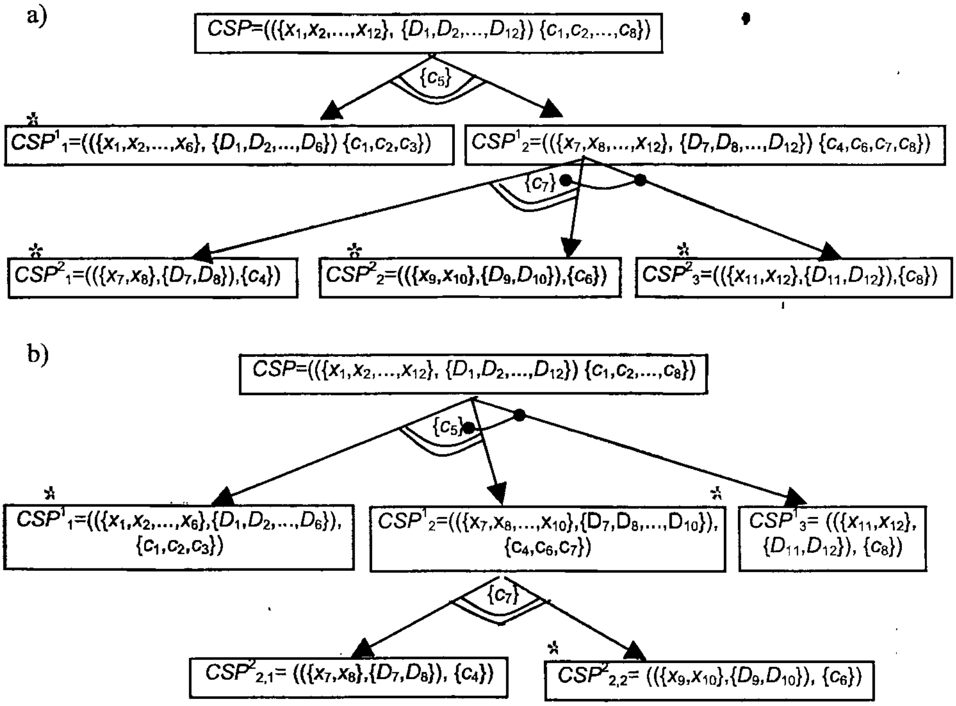
It's easy to notice, that problem formulated in such a way in natural decomposes into subproblems, in particular to elementary subproblems, which are not further decomposed.

2.2. Searching strategy prototyping

Let's consider the following problem: $CSP = ((X,D),C)$, where $X = \{x_1,x_2,\ldots,x_{12}\}$, $D = \{D_1,D_2,\ldots,D_{12}\}$, $C = \{c_1,c_2,\ldots,c_8\}$, and

$c_1 := P_1[x_1,x_2,x_3]$, $c_2 := P_2[x_2,x_4,x_5]$, $c_3 := P_3[x_4,x_6]$, $c_4 := P_4[x_7,x_8]$, $c_5 := P_5[x_4,x_7]$, $c_6 := P_6[x_9,x_{10}]$, $c_7 := P_7[x_8,x_9]$, $c_8 := P_8[x_{11},x_{12}]$.

Two, arbitrarily chosen, admissible decompositions of this problem are shown on fig.1.



Legend:

an AND'–like arc - solutions of distinguished problems depend on each other

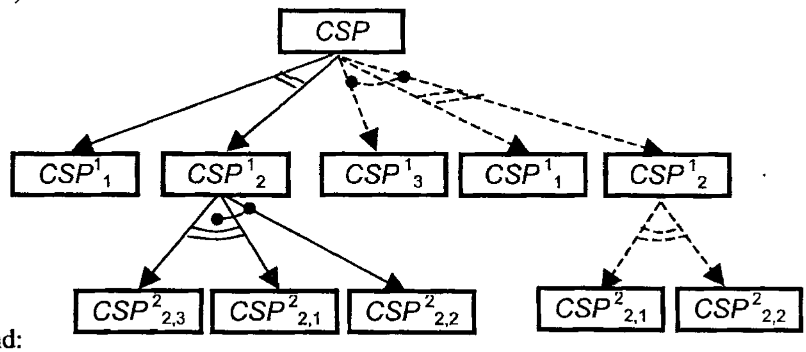
an AND"–like arc - solutions of distinguished problems are independent

* an elementary problem

Fig.1. Admissible problem decompositions CSP

Since each subproblem corresponds to a standard subproblem's structures, i.e., decision variables, domains and constraints, hence using an object-like modified AND/OR graph notation (see fig. 2) the analysis of all potential ways a CSP problem my be resolved, (i.e.

leads to admissible searching strategies encompassing the alternative orders of subproblems resolution) can be conducted.



Legend:

an AND'–like arc - solutions of distinguished problems depend on each other

an AND"–like arc - solutions of distinguished problems are independent

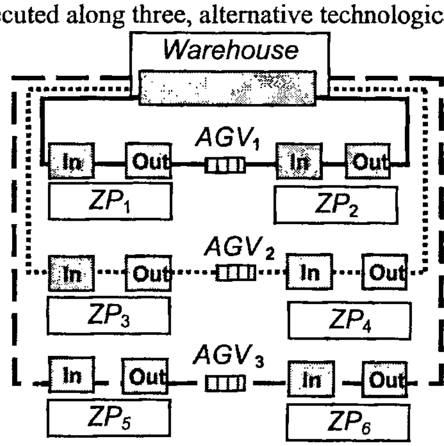
Fig. 2. AND/OR like graph of CSP decomposition

Note, that with arcs of AND/OR graph it is possible to bind weight factors determining the necessary number of searches, and in this way to chose strategy variant, e.g. with least number of backtrackings.

3. ILLUSTRATIVE EXAMPLE

Let us consider the production system composed of the set of resources ZP_i, operated by transportation means (AGV_j), warehouse, a set of automated guided vehicles (AGVs) as well as a set of workstations ZP_i equipped with input *In* and output *Out* buffers, respectively.

Assume the production order, concerning the production volume VZ=30, could be executed along three, alternative technological routes MP_1, MP_2, and MP_3.



Legend:
$ZP_1 - ZP_6$ - workstation
$ZT_1 - ZT_3$ – transportation means
Technological routes:
——— MP_1 (AGV_1)
.... MP_2 (AGV_2)
— — MP_3 (AGV_3)
In, Out – input-output buffers

Fig. 3. Diagram of a production system

Given are operation times and capacity of the input and output buffers (see Table 1). The capacity of the AGV_i is 5 details.

It is assumed that the AGVs operate along the following items:
AGV_1: storehouse, ZP_1, ZP_2
AGV_2: storehouse, ZP_3, ZP_4
AGV_3: storehouse, ZP_5, ZP

Table 1 Operating times and the buffers' capacity.

Workstation	Operation time	Buffer In	Buffer Out
ZP_1	2	5	5
ZP_2	1	5	5
ZP_3	1	7	7
ZP_4	1	7	7
ZP_5	1	6	6
ZP_6	2	6	6

It is assumed that each transportation operation takes one time unit. This period includes the loading and unloading times required. Also, the two time units elapse between two subsequent transportation operations is assumed. This period includes the passage time of an empty AGV and its relevant service time.

Moreover, it is assumed that due to a technical reasons the access to resources (ZP_3, ZP_4) is limited (from 11 to 15 time unit), and production can be realized along two out of three available production routes. The question regards: Whether the considered production order can be executed within the planning horizon $H = 75$ or not?

CSP specification. The problem considered consists of the following subproblems:

- related to production;
- routing (A)
- batching:
  - number of production batches (B)
  - size of production batches (E)
- scheduling (F)
- related to transportation (not decomposed into routing, scheduling and batching) (G).

A: Production routing

$$PSO_A = ((\{X_A\}, \{D_A\}), \{c_1, c_2\})$$

Decision variables:

$X_A = \{x_{A,1}, x_{A,2}, x_{A,3}\}$; $x_{A,i}$ – number of items produced along the i-th route .

Domains:

$D_A = \{d_{A,1}, d_{A,2}, d_{A,3}\}$; $d_{A,i} \in \{0, 5, 10, 13, 14, 15, 16, 17, 20, 25\}$

Constraints:

c_1: $\sum_{i=1}^{3} x_{1,i} = VZ$; $VZ = 30$ – volume of a production order.

c_2: $\sum_{i=1}^{3} sign(x_i) = 2$; where: $sign(x) = \begin{cases} 1 \text{ if } x > 0 \\ 0 \text{ if } x \le 0 \end{cases}$

B: Production batching (number of batches)

$PSO_B = ((\{X_B\}, \{D_B\}), \{c_3 \div c_5\})$

Decision variables:

$X_B = \{x_{B,1}, x_{B,2}\}$; $x_{B,i}$ – number of production batches following the i-th route.

Domains:

$D_B = \{d_{B,1}, d_{B,2}\}$; $d_{B,i} \in \{1..VZ\}$

Constraints:

c_3: $x_{B,i} \le vz_i$, i=1..2

c_4: $x_{B,i} \cdot W \ge vz_i$; vz_i – number of items produced according to the i-th route, W=5 – load capacity of AGV

c_5: $(x_{B,1} + x_{B,2}) \cdot W \ge VZ$

E: Production batching (batch size)

$PSO_E = ((\{X_E\}, \{D_E\}), \{c_6 \div c_9\})$

Decision variables:

$X_E = \{x_{E,1}, x_{E,2}, \ldots, x_{E,i}, \ldots, x_{E,L}\}$; $x_{E,i}$ – size of the i-th production batch, L – number of production batches, $L = x_{B,1} + x_{B,2}$

Domains:

$D_E = \{d_{E,1}, d_{E,2}, \ldots, d_{E,i}, \ldots, d_{E,L}\}$; $d_{E,i} \in \{1..VZ\}$

Constraints:

c_6: $\sum_{i=1}^{x_{B,1}} x_{E,1} = vz_1$, c_7: $\sum_{i=x_{B,1}+1}^{L} x_{E,1} = vz_2$

c_8: $\sum sign(x_{E,i}) = x_{B,1} + x_{B,2}$, c_9: $\sum_{i=1}^{L} x_{E,i} = VZ$

F: Production scheduling

$CSP_F = ((\{X_F\}, \{D_F\}), \{c_{10} \div c_{12}\})$

Decision variables:

$X_F = \{x_{F,1}, x_{F,2}, \ldots, x_{F,i}, \ldots, x_{F,K}\}$; $x_{F,i}$ – beginning the i-th production operation, $K = 2 \cdot (x_{B,1} + x_{B,2})$ – number of production operations

Domains:

$D_F = \{d_{F,1}, d_{F,2}, \ldots, d_{F,i}, \ldots, d_{F,K}\}$; $d_{F,i} \in \{1..H\}$; H=75 – planning horizon.

Constraints:

c_{10}: $\begin{cases} \forall i \in 2..x_{B,1}, & x_{F,i} > x_{F,i-1} + T_{F,i-1} \\ \forall i \in 1..x_{B,1}, & x_{F,i+L} \ge x_{F,i} + J_{F,i} \end{cases}$

$$c_{11}: \begin{cases} \forall i \in x_{B,1}+2..L, & x_{F,i} > x_{F,i-1} + T_{F,i-1} \\ \forall i \in x_{B,1}+1..L, & x_{F,i+L} \geq x_{F,i} + J_{F,i} \end{cases}$$

$T_{F,i}$ –operation time $T=f(x_{E,i})$; $J_{F,i}$ – unit production time increased by the minimum transportation time between working positions

$$c_{12}: \forall i \in \{x_{F,j}, \ldots, x_{F,j} + T_{F,j}\}, i \notin \{11, \ldots, 15\};$$

j – index of the operation executed on a resource with limited access

G: Transportation operations

$CSP_G = ((\{X_G\}, \{D_G\}), \{c_{13} \div c_{16}\})$

Decision variables:

$X_G = \{X_{G1}, X_{G2}, X_{G3}, X_{G4}, X_{G5}, X_{G6}, X_{G7}, X_{G8}, X_{G9}, X_{G10}\}$

$X_{G1} = \{x_{G1,1}, x_{G1,2}, \ldots, x_{G1,H}\}$

...

$X_{G10} = \{x_{G10,1}, x_{G10,2}, \ldots, x_{G10,H}\}$

$X_{Gi,j}$ – number of items transported (produced) along the i-th transportation (production) operation at the j-th moment,

H – planning horizon

Domains:

$D_G = \{D_{G1}, D_{G2}, D_{G3}, D_{G4}, D_{G5}, D_{G6}, D_{G7}, D_{G8}, D_{G9}, D_{G10}\}$

$D_{G1} = \{d_{G1,1}, d_{G1,2}, \ldots, d_{G1,H}\}$

...

$D_{G10} = \{d_{G10,1}, d_{G10,2}, \ldots, d_{G10,H}\}$

Transportation operations:

$d_{G1,i}, d_{G3,i}, d_{G5,i}, d_{G6,i}, d_{G8,i}, d_{G10,i} \in \{0..W\}$; where W – AGV load capacity

Production operations:

$d_{G2,i}, d_{G4,i}, d_{G7,i}, d_{G9,i} \in \{0..1\}$

Constraints:

$$c_{13}: \forall i = 1..(H-2), \begin{cases} x_{G1,i} \cdot x_{G3,i} \cdot x_{G5,i} \cdot x_{G1,i+1} \cdot x_{G3,i+1} \cdot x_{G5,i+1} \cdot x_{G1,i+2} \cdot x_{G3,i+2} \cdot x_{G5,i+2} = 0 \\ x_{G6,i} \cdot x_{G8,i} \cdot x_{G10,i} \cdot x_{G6,i+1} \cdot x_{G8,i+1} \cdot x_{G10,i+1} \cdot x_{G6,i+2} \cdot x_{G8,i+2} \cdot x_{G10,i+2} = 0 \end{cases}$$

$$c_{14}: \begin{cases} \forall i = 1..x_{B,1}, \forall j = 1..x_{E,i}, & x_{G2,x_{F,i}+(j-1)*J_{F,i}} = 1 \\ \forall i = 1..x_{B,1}, \forall j = 1..x_{E,i+x_{B,1}}, & x_{G4,x_{F,i+x_{B,1}}+(j-1)*J_{F,i+x_{B,1}}} = 1 \\ \forall i = 1..x_{B,2}, \forall j = 1..x_{E,i+2\cdot x_{B,1}}, & x_{G7,x_{F,i+2\cdot x_{B,1}}+(j-1)*J_{F,i+2\cdot x_{B,1}}} = 1 \\ \forall i = 1..x_{B,2}, \forall j = 1..x_{E,i+x_{B,1}+2\cdot x_{B,1}}, & x_{G9,x_{F,i+x_{B,2}+2\cdot x_{B,1}}+(j-1)*J_{F,i+x_{B,2}+2\cdot x_{B,1}}} = 1 \end{cases}$$

where: $J_{F,i}$ – unit time of an i-th production operation, , $H = 75$ planning horizon

$$c_{15}\text{: } \forall i = 2..H, \begin{cases} \sum_{h=1}^{i} x_{G2,h} \le \sum_{h=1}^{i-1} x_{G1,h} \\ \sum_{h=1}^{i} x_{G3,h} \le \sum_{h=1}^{i-1} x_{G2,h} \\ \sum_{h=1}^{i} x_{G4,h} \le \sum_{h=1}^{i-1} x_{G3,h} \\ \sum_{h=1}^{i} x_{G5,h} \le \sum_{h=1}^{i-1} x_{G4,h} \end{cases} \quad c_{16}\text{: } \forall i = 2..H, \begin{cases} \sum_{h=1}^{i} x_{G7,h} \le \sum_{h=1}^{i-1} x_{G6,h} \\ \sum_{h=1}^{i} x_{G8,h} \le \sum_{h=1}^{i-1} x_{G7,h} \\ \sum_{h=1}^{i} x_{G9,h} \le \sum_{h=1}^{i-1} x_{G8,h} \\ \sum_{h=1}^{i} x_{G10,h} \le \sum_{h=1}^{i-1} x_{G9,h} \end{cases}$$

The graphical representation of the CSP considered is shown in Fig. 4. Two of $|\{A,B,C,D,E\}|!$ possible ways of problem resolution are shown in Fig.5.

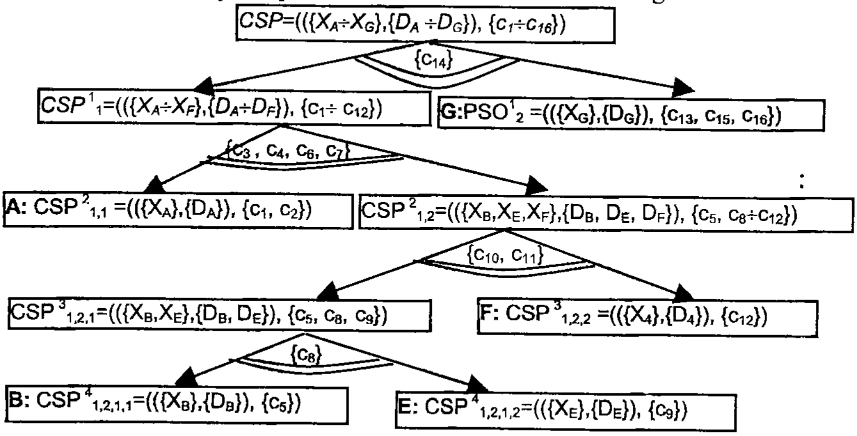


Fig. 4. The decomposition of CSP problem.

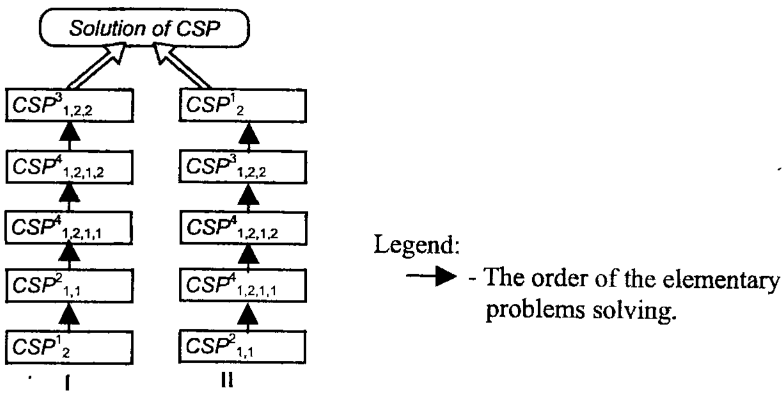


Fig. 5. The searching strategies.

Cost of using determined decomposition (i.e., the searching strategy) is estimated due to the following formulae.

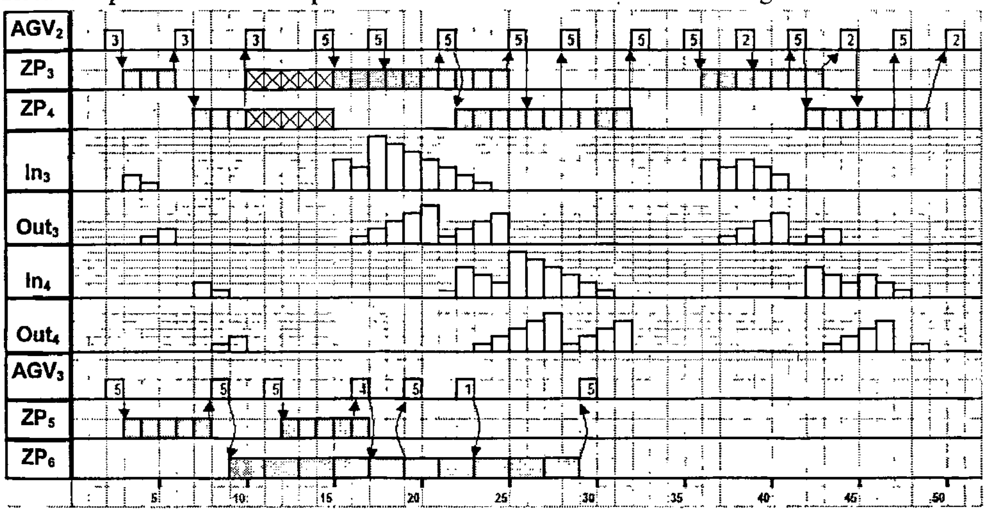
$$N = \sum_{i=1}^{n} \left(Z^{j_1}{}_1 \cdot Z^{j_2}{}_2 \cdot \ldots \cdot Z^{j_i}{}_i - 1 \right)$$

where n – number of subproblems

$Z^j i_i$ – the number of variables the $Z^j i$ problem resolved in the i-th order

So, the best strategy is the strategy I, where subproblems are resolved in order of their computational complexity increasing, i.e. in the order guaranteeing the lowest amount of backtrackings.

The strategy selected was implemented in the Oz language in Mozart system [6]. Resultant production flow is presented on the Gantt's chart shown in fig. 6.



Legend:

- [1] - transport operation (batch size);
- ⊠ - no access to resource;
- □ - a workpiece processing time;
- □ - amount of storaged workpieces.

Fig . 6. Admissible solution of production flow in Gantt's chart

The example presented illustrates the way the reference model permits to perform the analysis of admissible searching strategies to solve the problem of flow production planning. The strategy selected is an optimal one in the sense of a lowest number of potential backtrackings required in order to exam the all possible substitutions of decision variables.

From the example provided it follows also that the CP based framework provides a promising perspective for a new software tools enabling one to cope with such kind of tasks as a production flow planning in an on-line mode.

4. CONCLUDING REMARKS

Despite of many problems regarding the small and medium size enterprises (SME) management the main question is how to be able to respond whether capability of a firm at hand can be enough to accept a new production order? How to obtain such a response in an on-line mode? What mean of production order processing is the most efficient one?

The software tools available provide, however only costly and time-consuming potential to exam few arbitrary assumed versions of work order processing. That is because of combinatorial explosion of possible solutions caused by possible technologies and tools assignment, material handling, transportation and storage facilities assignments, production and transportation lot-sizing, scheduling and pricing, and so on. It means that tools enabling one to cope with such kind of tasks in an on-line mode are of crucial importance. This need implies requirements for the new approaches and paradigms.

The promising perspective seems to be based on CP based framework especially in the context of the reference model presented which allows to make analyses of admissible searching strategies to solve problems of flow production planning. Moreover, it enables to estimate the number of decision variables domains values substitution, i.e., to choose the best sequence of elementary subproblems solution (in the sense of a number of potential backtrackings).

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